Meadows or Malls?

Introduction

Durango has come into possession of some new land, given by three different organizations. The Dalton gang disbanded and in doing so left behind their 300 acre ranch to the city, the U.S military had given away their 100 acre base entitled the old fort, and finally the Boston mining company had given away 150 acres. All of this land was given to the city with no restrictions on its use, meaning that the city could do whatever they wanted with it.

Altogether there was 550 acres of land to use, however there were two major splits in the population that argued on how the land should be used. The first group which will be appropriately titled the business group which argued for the development of the land to be purely residential and business related. The second group entitled the recreation group wanted the land to be used for recreational purposes only. This triggered a debate between the two groups leading to the business sector to take the first victory, this gave business at least 300 acres of land for development. Though environmental and recreation minded parties felt like the more attractive land should be for recreational usages.

Finally, a compromise was found between the two group and it involved:

That 200 acres of military and mining land can be used for recreation

The amount of army base land used for recreation and the amount of Dalton ranch land used for development together had to total exactly 100 acres.

People were also informed that when it came to providing power and water, just the general utilities. The people of Durango wanted to keep the cost to the city to a minimum, this is the graph that depicts the pricing:

Parcel	Improvement costs per acre for recreation	Improvement costs per acre for development
Dalton Ranch	\$50	\$500
The Old Fort	\$200	\$2,000
The Boston Mine	\$100	\$1,000

What we have to do in this scenario is to establish a good layout of the land, that works with the compromises and uses all the land.

The first step to understanding how to set up the equation is putting the different types of land we have into variables. The best way of going about it is dividing the land into the land + the type of development that will be happening on it. An example of this would be R which represents Dalton Ranch plus adding the type development which in this case would be Recreation so it can be represented by R_R and this represents Dalton Ranch Recreation. If this is the way that we can show the variables then it is safe to write them all in that way.

Variables

D_R
D_D (D is for development, R represents recreation)
A_R
A_D
M_R
M_D

So, now we have our variables listed out and we can begin to represent our constraints in a mathematical way. We know that Development has been given at least 300 acres of land in general so we can write an equation that looks like this to represent it $D_D + A_D + M_D \geq 300$, what this means is that all development spread across all the different land must be equal to or more than 300 acres. Another equation that demonstrates our constraints is that 200 acres of military/mining land can go to recreation this can be written as $M_R + A_R \leq 200$. The final constraint that demonstrates what land can go to is that The amount of army base land used for recreation and the amount of Dalton ranch land used for development together had to total exactly 100 acres. This can be written as $A_R + D_D = 100$. That is the final restriction on land use, there are 3 other restrictions that simply state that the land that is being used cannot be negative amounts as that in this scenario is impossible because land cannot become negative as far as I know. I will now list the constraints to make it easier to read and group together.

Constraints

1. $D_D + A_D + M_D \geq 300$
2. $M_R + A_R \leq 200$
3. $A_R + D_D = 100$
4. $D_R + D_D \geq 0$
5. $A_R + A_D \geq 0$
6. $M_R + M_D \geq 0$
7. $R_R + R_D = 300$
8. $A_R + A_D = 100$
9. $M_R + M_D = 150$

System of linear equations

We now have to set up the systems of linear equations that we will be using to later plug into our matrices which will be the eventual solving step of our problem. Systems of linear equations are a grouping of equations that have the same variables that are working together to solve a bigger piece. It can be two equations or in our case six. Our systems will be the different combinations of constraints that we have, we set this up by combining the important constraints that we have in different ways and with other combinations. Some combinations will not work with the final answer or some of the constraints but that is fine as we have other combinations that have the potential to work.

List of the combinations of constraints

- I, II, III, VI, IV, V
- I, II, III, VI, IV, VIII
- I, II, III, VI, IV, IX
- I, II, III, VI, IV, XII
- I, II, III, VI, V, VIII
- I, II, III, VI, V, IX
- I, II, III, VI, V, XII
- I, II, III, VI, VIII, IX
- I, II, III, VI, VIII, XII
- I, II, III, VI, IX, XII

Matrices

A matrix is a different way of setting up an equation typically we are used to seeing addition subtraction equations written like this:
5 + 5 = 10 or 20 - 4 = 16
But, when it comes to matrices it would be represented as this

$$A = \begin{bmatrix} 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 10 \end{bmatrix}$$

But, this is a very simple equation. What would happen if there were a lot more numbers that needed to be added up?

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix}$$

$$B = \begin{bmatrix} 8 & 9 & 10 \\ 11 & 12 & 13 \\ 14 & 15 & 16 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 11 & 13 & 15 \\ 17 & 19 & 21 \\ 23 & 25 & 27 \end{bmatrix}$$

What this demonstrates is that when working with matrices of the same dimensions the numbers in each spot correspond to the numbers in the same spot. So when adding, take the

first number in the left hand corner in both matrices and add them together and as we see it is basically just saying 3 + 8 = 11 and this is the same for all numbers. This method is practically just adding everything together at once.
Let's move onto multiplication with matrices which gets a little more complicated. First though I will add another reference image:

$$A = \begin{bmatrix} 7 & 3 & 4 \\ 6 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 \\ 13 & 7 \\ 12 & 8 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 94 & 74 \\ 44 & 40 \end{bmatrix}$$

You see how the numbers are much larger than you would think if you happened to be multiplying straight on such as 7 x 1 = 7 which is not the case here. Instead it multiplies the top row by the first column then adds up the sum of the multiplication to equal the whole. Then that is repeated by multiplying the first row again but by the other column. Then follows the same process for the bottom row. So it looks like this:

7 x 1 + 3 x 13 + 4 x 12 = 94 7 x 3 + 3 x 7 + 4 x 8 = 74
6 x 1 + 2 x 13 + 1 x 12 = 44 6 x 3 + 2 x 7 + 1 x 8 = 40

This is why the sum doesn't come out as a 3 x 2 or 2 x 3 matrix because there are only four sums.

So how does this help us solve our problem? What does these silly numbers and lines have to do with the fair distribution of land? Well let me explain. There is a way to place our constraints into the matrix calculator and solve it that way instead of having to solve multiple systems at once. This makes it significantly easier because there is time you have to wait solving and setting up equations only to find out that it has no answer.

$$A = \begin{bmatrix} 4 & 8 \\ 6 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -0.45 \\ 1.1 \end{bmatrix}$$

This is called using the inverse of A to find out what X and Y are, the reason that you cannot see X and Y is because it does not work with the calculator because x and y are not able to be used as a variable. So, what you have to do is imagine X and Y is there but unseeable so what this problem would look like is:

4x + 8y = 7
6x + 8y = 5

When you inverse matrix A it gives you something called the multiplicative inverse and you are then able to times matrix B by it to get what Y and X is equal to.

It is becoming clearer to see how this will be used to make our combinations easier to solve.

Now that we have the basics down we can then go on to solve the major problem that has been hanging over our heads.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 300 \\ 100 \\ 150 \\ 100 \\ 300 \\ 0 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 225 \\ 75 \\ 25 \\ 75 \\ 0 \\ 150 \end{bmatrix}$$

This is the method to solve the final problem. The first 3 columns represent constraints 7-9 which is the amount of land we have which can be seen in matrix B, then constraint 3 which can be seen in the 4th row of matrix B. Constraint 1 which can be seen in the 5th row of matrix B. We can then do the inverse and multiply matrix B by the multiplicative inverse and there is the final answer.

Why this answer though? Well for one it satisfies all the given constraints and secondly it is the cheapest option out of them all. Of course I will not run down every combination of constraints in this methodical way but here is a very helpful graph.

Systems	R_R	R_D	A_R	A_D	M_R	M_D	Notes
I, II, III, VI, IV, V	50	250	-150	250	350	-200	Fails Positivity Constraint
I, II, III, VI, IV, VIII	200	100	0	100	50	100	Total Cost: 365,000
I, II, III, VI, IV, IX	225	75	25	75	0	150	Total Cost: 353,750
I, II, III, VI, IV, IX	150	150	-50	150	150	0	Fails Positivity Constraint
I, II, III, VI, V, VIII	200	100	0	100	300	-150	Fails Positivity Constraint
I, II, III, VI, V, IX	400	-100	200	-100	0	150	Fails Positivity Constraints
I, II, III, VI, V, XII	250	50	50	50	150	0	Fails Constraint IV
I, II, III, VI, VIII, IX	200	100	0	100	0	150	Total Cost: 410000
I, II, III, VI, VIII, XII	200	100	0	100	150	0	Fails Constraint IV
I, II, III, VI, IX, XII							Not Possible

You can see here that a vast majority of them have negative numbers which fails the positivity constraints that we have set in place in the beginning. Or it just happens to fail a given constraint like constraint 1 which makes it so that development must be at least 300 acres so with this you cannot use this combination. Finally, it is the cheapest option coming out to be 353,750$. This makes system three the best option for the city to use.

Reflection

This unit felt slightly better than the last and I am beginning to get used to the pace and routine that the classes curriculum takes. Though the more we do things the more I long for that absolute traditional curriculum that my old high school taught, the one where we do the work till we understand the functions and how to do it. Not saying that it's the best way of going about things or even a good way it just happened to make more sense and I never had this feeling of lost or confused because there was always practice and not the constant exchange of new topics that we have in this class. That isn't to say this is bad though, I really liked the use of matrices and I feel like it is something that I can use later on down the road and will probably come in handy. The learning process of matrices made sense to me as well which was a great change of pace as opposed to how it usually is with me being just completely thrown for a loop, but for once it was intuitive I am sure you can relate to that feeling when something is just simple to you. That is why I used it to solve the unit problem as opposed to other ways, because with things like three variable linear equations I would set them up properly and all that but it would come out wrong and I could redo it and redo it but for some reason unbeknownst to me or even god it was wrong. Predestined to be wrong is what it seemed like it was, awful.

But thank god there were matrices, what a great and lovely thing in a sea of confusing equations. Another thing that I took up during this unit was asking people how to do certain things, by people I mean my peers. Which is something that I didn't really do last semester probably because I was less comfortable with the people around me but now I feel like I can talk to almost anyone about how to do certain things. So has it been a good unit? Half and Half I would have been destroyed if matrices weren't introduced but I also learned how to talk and ask questions. In the long run this has been a good unit, but there were times that I felt like slamming my head into the table.